Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

On the Theoretical Justification of Ibrahim's Method

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Introduction

IBRAHIM'S method has been comprehensively described by Ibrahim and his associates.[1-3] Central to the application of the method is the inversion of a nominally singular matrix. The rank of the matrix is completed by noise in the data.

In justifying the method,[1] Ibrahim uses two different (and conflicting) characterizations for the noise. This conflict will be illustrated here and its implications considered.

It is perhaps worthwhile to note the simultaneous work done by Dunn,[4] where the analysis is in many ways similar.

Basis of the Method

The free vibration problem is characterized by

$$AX = \dot{X} \tag{1}$$

where A is a matrix $(2n \times 2n)$ describing the structure of the system and X and $\dot{X}$ are observations of the state vectors of the system. These may be chosen square.

If X is nonsingular, then A may be uniquely determined

$$A = \dot{X}X^{-1} \tag{2}$$

In all realistic experimental conditions, however, each observation vector x_i (column of X) will only be composed of a small number $2m$ of the modal vectors of the system.

Enough information for subsequent analyses could be extracted by using the Moore-Penrose generalized inverse X^g (see, e.g., Wilkinson and Reinsch[5]) instead of X^{-1} (which does not exist).

The procedure adopted by Ibrahim, however, differs markedly from this. Completion of the rank of X is provided by noise on the data. This is characterized by Eq. (14) of Ref. 3:

$$x = \sum_{k=1}^{2m} p_k e^{\lambda_k t} + n(t) \tag{3}$$

i.e., the observation vector x is composed of the sum of $2m$ normal modes with initial excitation p_k which decay (in passive systems) exponentially by the factor $e^{\lambda_k t}$ and a vector of noise $n(t)$. Whereas the elements of $n(t)$ may be partially correlated due to characteristics of the experimental setup, it generally would be assumed that these elements were independent random variables.

In subsequent discussion, however, Ibrahim uses a characterization [Ref. 1, Eq. (15)],

$$x(t) = \sum_{k=1}^{2m} p_k e^{\lambda_k t} + \sum_{k=2m+1}^{2n} N_k e^{r_k t} \tag{4}$$

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and states that, "the extra degrees of freedom act as outlets for the noise."

Identification of Inconsistencies

The matrix X is required to be nonsingular for the method to work. The true modes of the system $\{p_k\}$ contribute $2m$ linearly independent (and additionally orthogonal) vectors to a basis to span X. Consequently, the remaining noise vectors defined by Ibrahim $\{N_k\}$ must contribute a further $(2n-2m)$ linearly independent vector to complete the rank of X. It is not necessary for the vectors N_k to be mutually orthogonal, nor in fact is it necessary for the subspace $\{N_k\}$ to be orthogonal to the subspace $\{p_k\}$.

Orthogonality of the two subspaces would, however, be necessary to justify Ibrahim's assertion, "The qualitative explanation for this situation is that the extra degrees of freedom act as outlets for the noise."

This characterization of noise is inconsistent with the usual characterization of noise, where each element of X is perturbed by an independent random variable. This will now be demonstrated.

Define X to be composed of the sum of two matrices:

$\boldsymbol{D}$: deterministic matrix spanned by $\{p_k\}$
$\boldsymbol{R}$: random matrix spanned by $\{N_k\}$
X: $\boldsymbol{D}+\boldsymbol{R}$

$\boldsymbol{R}$ has rank $(2n-2m)$ since it is spanned by the linearly independent $\{N_k\}$. It is possible by permutation to ensure that the leading $(2n-2m)\times(2n-2m)$ square of $\boldsymbol{R}$ is nonsingular; denote the columns of $\boldsymbol{R}$ by $\{r_k\}$. Taking the first $(2n-2m)$ elements of $r_{2n-2m+1}$, these may be expressed as a linear combination of the corresponding elements of the previous $(2n-2m)$ columns without compromising their property of randomness. However, the rank limitation requires that the succeeding $(2n-2m+1)$th elements of $r_{2n-2m+1}$ be fixed by the same linear combination. Thus, the rank limitation implies that this element is fixed by the previous elements of the vector, thereby infringing on the requirement for independence of the random variable.

Correct Characterization of the Noise Matrix

It may be shown inductively, from the above analysis, that any noise representation based on the usual model of independent random variables requires a matrix $\boldsymbol{R}$ of full rank. Such a matrix has the inherent effect of corrupting observations of the true modes of the system. While this would conform to the intuitive assessment of most analysts, it conflicts with Ibrahim's suggestion that the additional degrees of freedom act, "as outlets for the noise."

It should be noted that Dunn[4] notes the rather surprising characteristic of his method that mode shapes may be correctly identified while the corresponding identified frequency appears grossly in error. In this case two of Ibrahim's measures would give conflicting results—(the Modal Confidence Factor and the Mode Shape Correlation Constant, which is inappropriately styled as a constant).

Conclusion

The noise characterizations repeatedly used by Ibrahim do not conform to the generally understood definition of random noise. The acceptance of such erroneous assumptions concerning noise will not lead to acceptance of incorrect modes, but may give rise to the rejection of genuine structural modes.

References

[1]Ibrahim, S.R., "Computation of Normal Modes from Identified Complex Modes," *AIAA Journal,* Vol. 21, March 1983, pp. 446-451.

[2]Ibrahim, S.R. and Papa, R.S., "Large Modal Survey Testing Using the Ibrahim Time Domain Identification Technique," *Journal of Spacecraft and Rockets,* Vol. 19, Sept.-Oct. 1982, pp. 459-465.

[3]Pappa, R.S. and Ibrahim, S.R., "A Parametric Study of the Ibrahim Time Domain Modal Identification Algorithm," *Shock and Vibration Bulletin,* Vol. 51, May 1981, pp. 43-72.

[4]Dunn, W.H., "Modal Analysis of Transient Response Data Using a Complex Exponential Algorithm," National Engineering Laboratory, East Kilbride, Scotland, N.E.L. Rept. 641, Aug. 1977.

[5]Wilkinson, J.H. and Reinsch, C., *Handbook for Automatic Computation, Vol. 2: Linear Algebra,* Springer Verlag, Berlin, 1972.

Reply by Author to J.A. Brandon

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EQUATIONS (1) and (2) in Brandon's Comment are not the basis for the Ibrahim Time Domain (ITD) Method. These equations are only used in the special case of computing normal modes from measured complex modes. The ITD identification algorithm, however, uses Eq. (3) as the mathematical model for the identification of modal parameters.

According to the Comment, Brandon's concern is the validity of the theory of oversized identification model reported in his Ref. 2.

Such concern is based on the necessary orthogonality requirement between the noise modes subspace $\{N_k\}$ and structural modes subspace $\{p_k\}$. The fact is that these two subspaces will be orthogonal. The vectors $\{N_k\}$ are neither defined nor unique, but they are merely computational modes. If the matrix A is successfully computed, and since the set of vectors $\{p_k\}$ and $\{N_k\}$ are both the eigenvectors of one matrix, they will come out to be mutually orthogonal.

Engineering applications of mathematics are usually based on assumptions that are later tested for validity. The ones used in the theory of the oversized identification model, after extensive testing, have proved to be sound ones. In Brandon's Ref. 2, a two-degree-of-freedom system, with the severe simulated conditions of extremely low levels of measurements noise and ill conditioning, was successfully identified using a 300-degree-of-freedom identification model.

Perhaps a supplement to Brandon's Eq. (4) stating that "where the subspace $\{N_k\}$ is assumed to be orthogonal to the subspace $\{p_k\}$...," would have obviated this debate.

Using the same reasoning as in Brandon's explanation with matrices D and R, it also could have been as easily assumed that R has a rank of $2n$ rather than $(2n-2m)$. The vectors r_k can be expressed as:

$$r_k = \sum_{i=1}^{2n} W_i f_i(t_k)$$

where $\{W_i\}$ is linearly independent. In such a case $\{W_k\}$ and $\{N_k\}$ will be related by:

$$\sum_{i=1}^{2n} W_i f_i(t_k) = \sum_{i=1}^{2n-2m} N_i e^{r_i t_k}$$

Before introducing the concept of the oversized identification model, time domain identification of modal parameters of structures proved to be unfeasible. This was due to the inability to precisely determine the number of modes contributing to the responses and to easily deal with high levels of noise. The concept under discussion offers a direct approach to solving these two problems. To the best of my knowledge, this concept is being used in most of the newly developed time domain identification algorithms.

In closing, I would like to express my sincere thanks to Dr. Brandon for his interest and for allowing me the opportunity to try to clarify this issue.

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Addendum: "Generalized Dynamic Reduction in Finite Element Dynamic Optimization"

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WE would like to call the reader's attention to the fact that Generalized Dynamic Reduction is a novel concept introduced by Dr. Richard H. MacNeal of the MacNeal-Schwendler Corp., as mentioned in Ref. 3 and in the backup paper. The following references should be added to this Synoptic.

[6]"MSC/NASTRAN Application Manual," The MacNeal-Schwendler Corp., Los Angeles, Calif., April 1981, pp. 2.4 1-2.4.39.

[7]"Handbook for Dynamic Analysis," The MacNeal-Schwendler Corp., Los Angeles, Calif., 1983, pp. 4.1.5-4.1.13.

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Errata: "The Two-Dimensional Laminar Wake with Initial Asymmetry"

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[AIAA **21**, pp. 1347-1348 (1983)]

OMISSION of certain parentheses has distorted two equations in this paper. Equation (6) should read:

$$\bar{u} \equiv 1 - \frac{u}{u_e} = \bar{u}(x,y;M_e,T_w/T_{oe},P)$$

$$= \frac{1}{2}\Big\{\exp[(P+1)^2x + (P+1)y]$$

$$\times \left[1 - \mathrm{Erf}\left((P+1)\sqrt{x} + \frac{y}{2\sqrt{x}}\right)\right]$$

$$+ \left(\exp\left[\left(\frac{P+1}{P}\right)^2 x - \left(\frac{P+1}{P}\right)y\right]\right)$$

$$\times \left[1 + \mathrm{Erf}\left(\frac{y}{2\sqrt{x}} - \frac{P+1}{P}\sqrt{x}\right)\right]\Big\}$$

and Eq. (8) should read:

$$\bar{u}(y=0) = e^{4x}(1 - \mathrm{Erf}2\sqrt{x})$$

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